New Physics from Galaxy Clustering III

Based on arXiv.2411.tomorrow
done in collaboration with M. Ballardini (UniFE)

Refining the nonlinear modelling of primordial oscillatory features

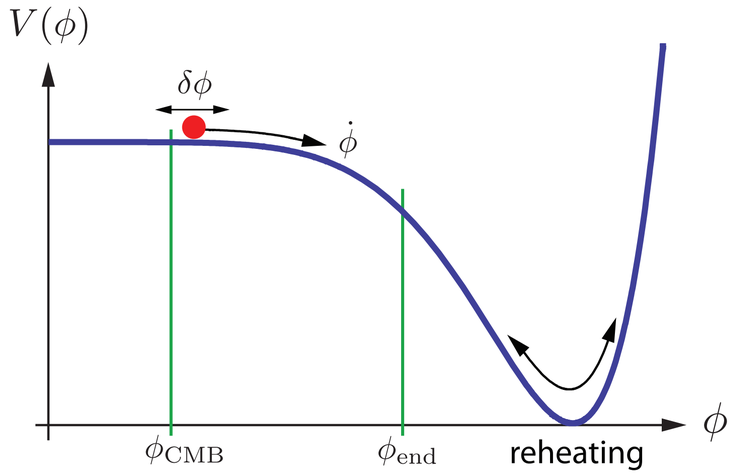
NICOLA BARBIERI (UNIFE) – 04/11/2024

Motivation: theory side

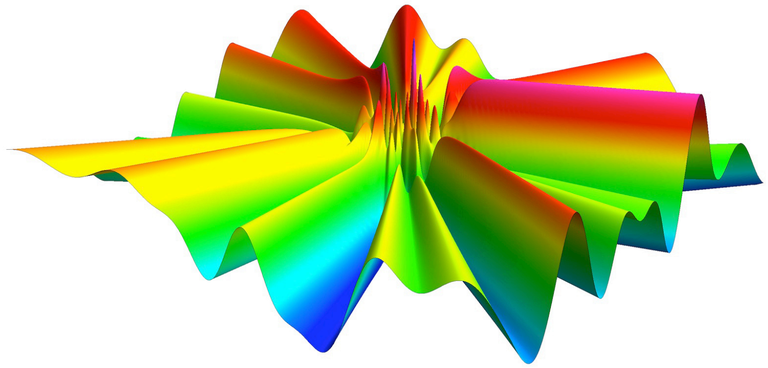
Single-field slow roll inflation is (most probably) a simplified model:

- It assumes a single-field slowly rolling down its potential;
- It assumes all other fields decoupled during inflation;
- Assumes a minimal coupling to gravity.

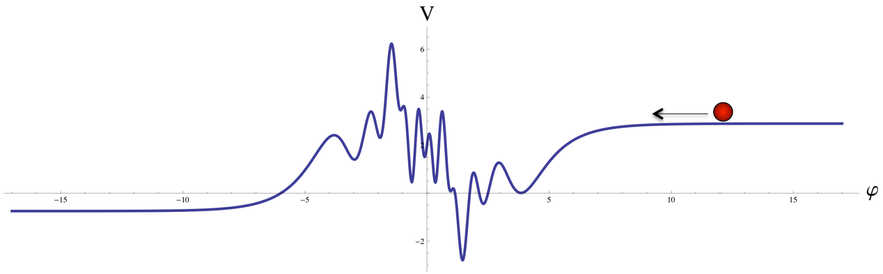
Single-field slow roll inflation typical potential



[Credits: D. Baumann]



[Credits: R. Kallosh & A. Linde]

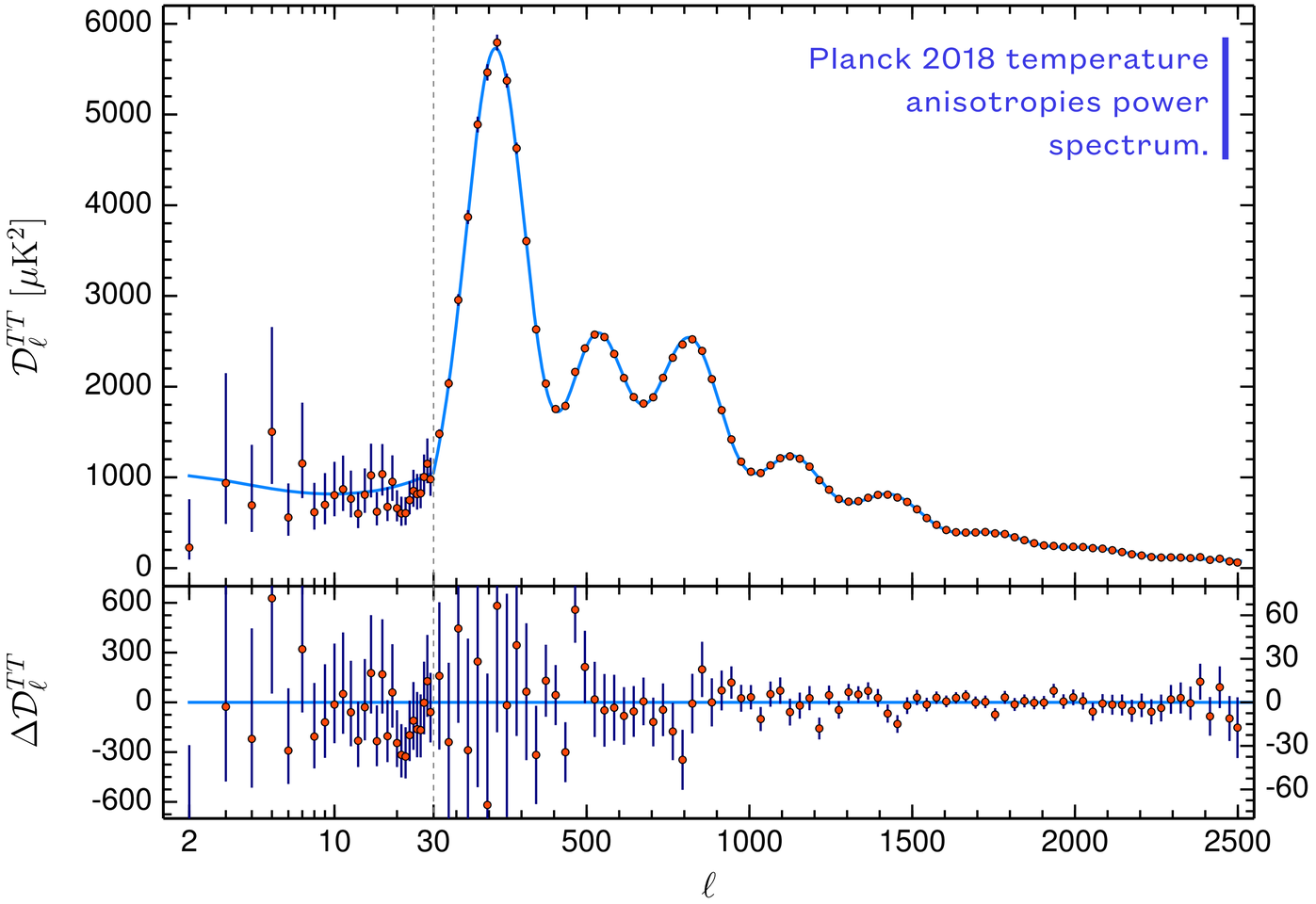


[Credits: R. Kallosh & A. Linde]

Motivation: observational side

CMB temperature power spectrum show anomalies which can be related to features in the primordial power spectrum

[Credits: Planck collaboration]

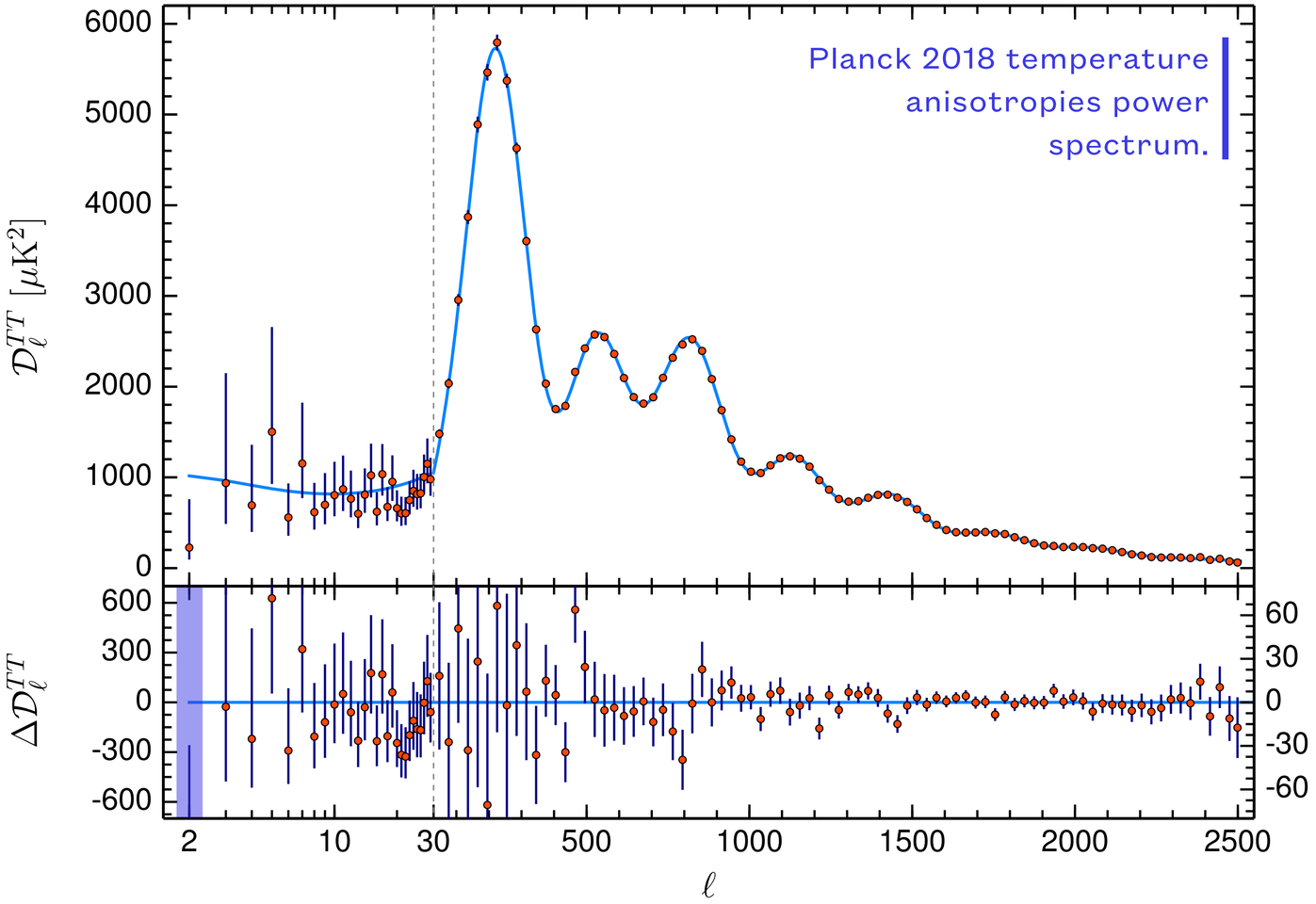


Motivation: observational side

CMB temperature power spectrum show anomalies which can be related to features in the primordial power spectrum

- Quadrupole anomaly
- Low multipoles deficit

[Credits: Planck collaboration]

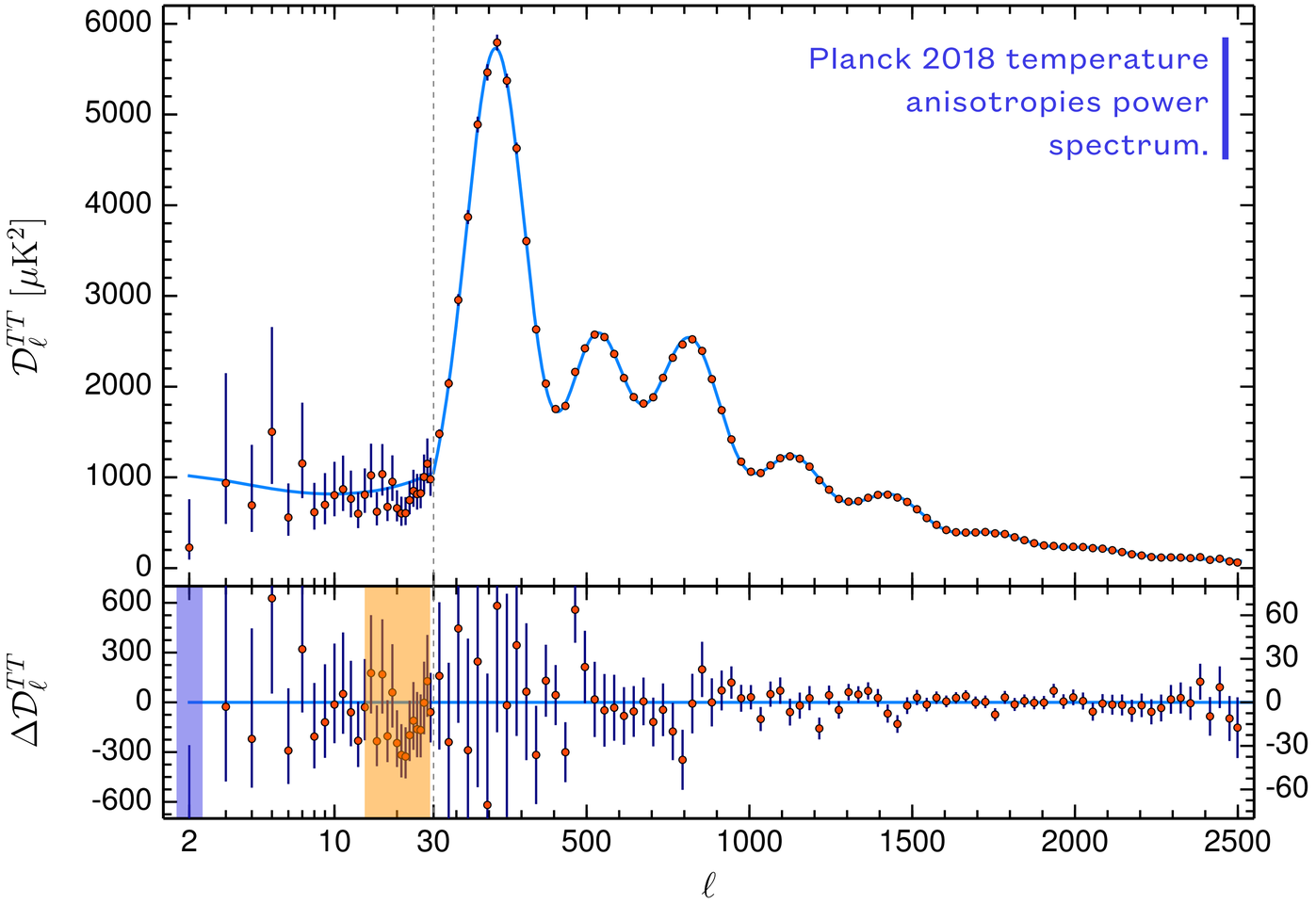


Motivation: observational side

CMB temperature power spectrum show anomalies which can be related to features in the primordial power spectrum

- Quadrupole anomaly
- Low multipoles deficit
- Low multipoles dip

[Credits: Planck collaboration]

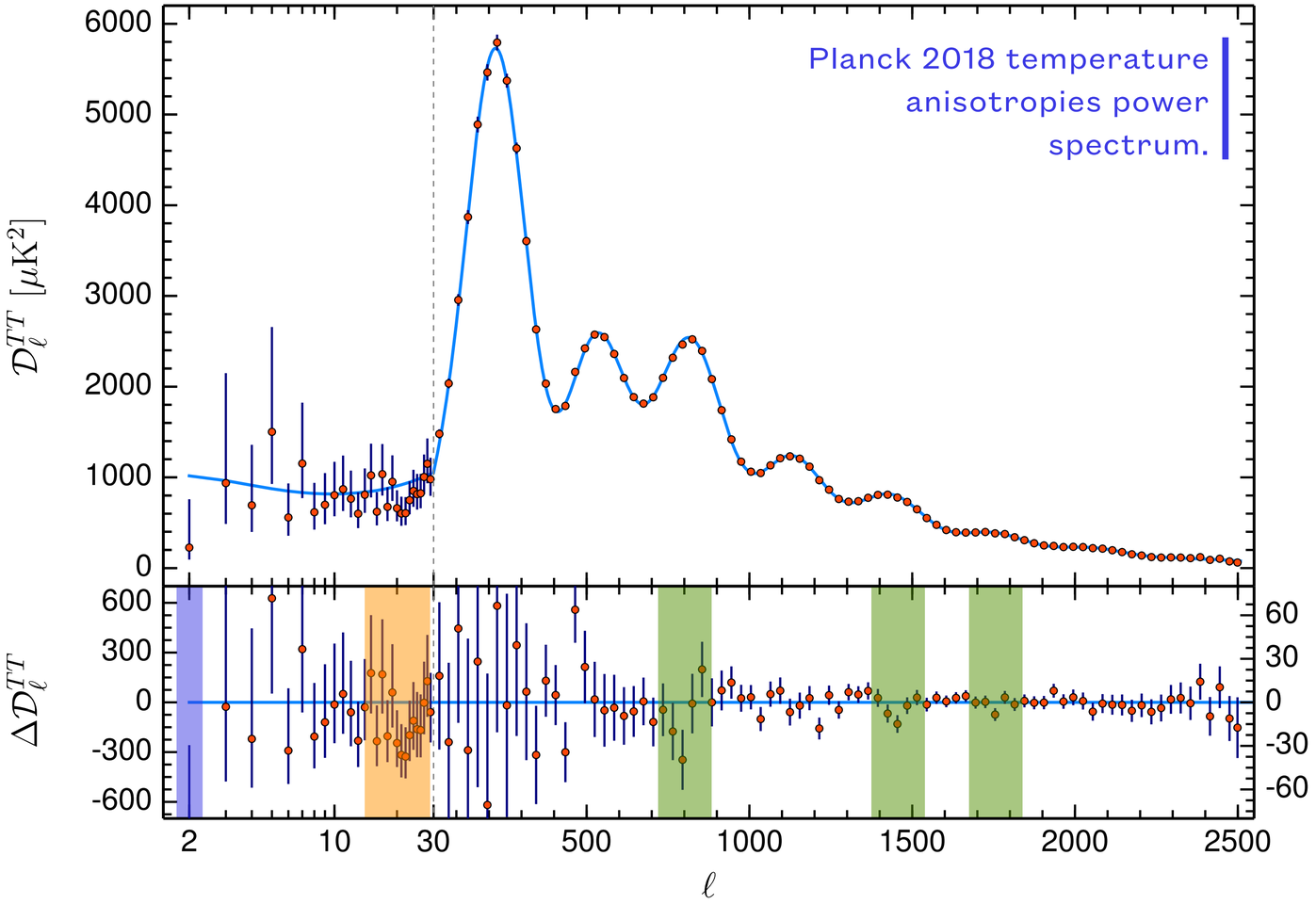


Motivation: observational side

CMB temperature power spectrum show anomalies which can be related to features in the primordial power spectrum

- Quadrupole anomaly
- Low multipoles deficit
- Low multipoles dip
- Several features at small scales…

[Credits: Planck collaboration]



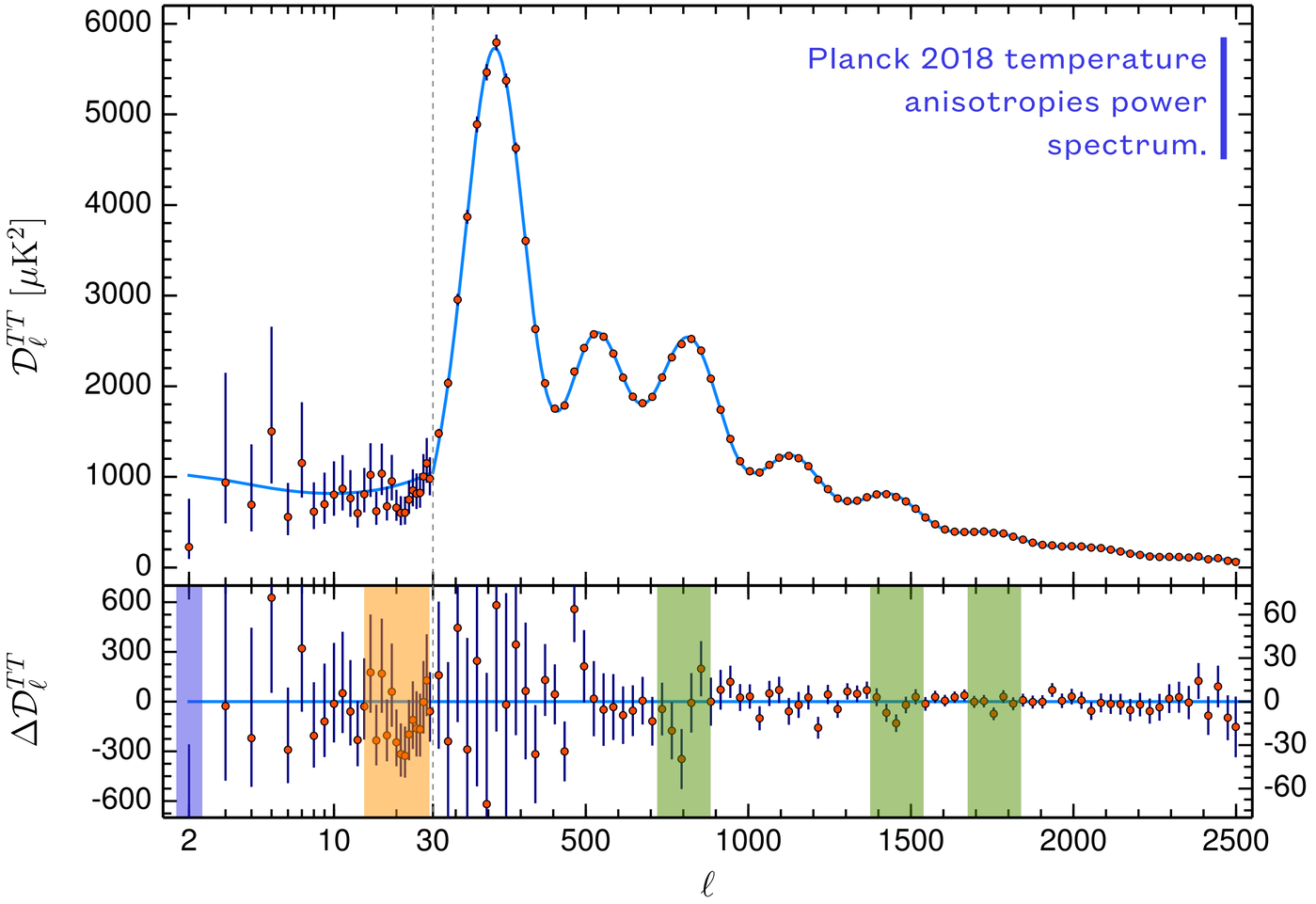
Motivation: observational side

CMB temperature power spectrum show anomalies which can be related to features in the primordial power spectrum

- Quadrupole anomaly
- Low multipoles deficit
- Low multipoles dip
- Several features at small scales…
- Other anomalies such as “lensing excess”

[Domenech & Kamionkowski JCAP **11** (2019) 040]
[Ballardini & Finelli JCAP **10** (2022) 083]
[Antony et al., PRL **130** (2023) 11]

[Credits: Planck collaboration]



Methodology: TSPT

[Blas et al., JCAP **07** (2016) 052]
[Blas et al., JCAP **07** (2016) 028]
[Vasudevan et al., JCAP **09** (2019) 037]

Idea: evolve the statistical distribution function of the fields rather than the fields themselves.

In analogy with particle physics, we can approach the problem with a path integral formulation:

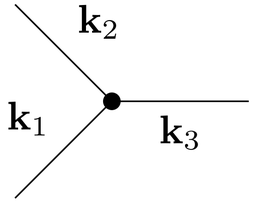
$$Z\left[J;\eta\right]=\int\mathcal{D}\Theta_{\eta}\,\mathcal{P}\left[\Theta_{\eta};\eta\right]\exp\left\{\int\mathrm{d}^{3}k\,\Theta_{\eta}\left(\mathbf{k}\right)J\left(-\mathbf{k}\right)\right\}$$

Methodology: TSPT

[Blas et al., JCAP **07** (2016) 052]
[Blas et al., JCAP **07** (2016) 028]
[Vasudevan et al., JCAP **09** (2019) 037]

Idea: evolve the statistical distribution function of the fields rather than the fields themselves.

In analogy with particle physics, we can approach the problem with a path integral formulation:



$$= -\frac{1}{g^2(\eta)}\frac{1}{3!}\bar{\Gamma}_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$\mathbf{k} \quad = \quad g^2(\eta)\,P_L(k)$$

TSPT diagrammatic expansion

$$Z[J;\eta] = \int \mathcal{D}\Theta_\eta\, \mathcal{P}[\Theta_\eta;\eta] \exp\left\{\int \mathrm{d}^3k\, \Theta_\eta(\mathbf{k})\, J(-\mathbf{k})\right\}$$

- **Correlators** can be computed through functional derivatives

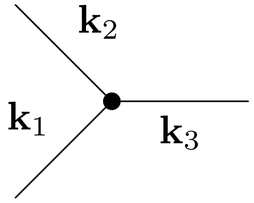
$$\langle \Theta_\eta(\mathbf{k}_1)\cdots\Theta_\eta(\mathbf{k}_n)\rangle = \left.\frac{\delta^n Z[J;\eta]}{\delta J(-\mathbf{k}_1)\cdots\delta J(-\mathbf{k}_n)}\right|_{J=0}$$

Methodology: TSPT

[Blas et al., JCAP **07** (2016) 052]
[Blas et al., JCAP **07** (2016) 028]
[Vasudevan et al., JCAP **09** (2019) 037]

Idea: evolve the statistical distribution function of the fields rather than the fields themselves.

In analogy with particle physics, we can approach the problem with a path integral formulation:



$$= -\frac{1}{g^2(\eta)}\frac{1}{3!}\bar{\Gamma}_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$\mathbf{k} \quad = \quad g^2(\eta)\,P_L(k)$$

TSPT diagrammatic expansion

$$Z[J;\eta] = \int \mathcal{D}\Theta_\eta\, \mathcal{P}[\Theta_\eta;\eta] \exp\left\{\int \mathrm{d}^3k\, \Theta_\eta(\mathbf{k})\, J(-\mathbf{k})\right\}$$

- **Correlators** can be computed through functional derivatives

$$\langle \Theta_\eta(\mathbf{k}_1)\cdots\Theta_\eta(\mathbf{k}_n)\rangle = \left.\frac{\delta^n Z[J;\eta]}{\delta J(-\mathbf{k}_1)\cdots\delta J(-\mathbf{k}_n)}\right|_{J=0}$$

- **Temporal evolution** is encoded in Liouville's equation for the conservation of probability

$$\frac{\partial}{\partial\eta}\mathcal{P}[\Theta_\eta;\eta] + \int \mathrm{d}^3p\, \frac{\delta}{\delta\Theta_\eta(\mathbf{p})}\left(\mathcal{I}[\Theta_\eta;\mathbf{p},\eta]\,\mathcal{P}[\Theta_\eta;\eta]\right) = 0$$

Methodology: TSPT

[Blas et al., JCAP **07** (2016) 052]
[Blas et al., JCAP **07** (2016) 028]
[Vasudevan et al., JCAP **09** (2019) 037]

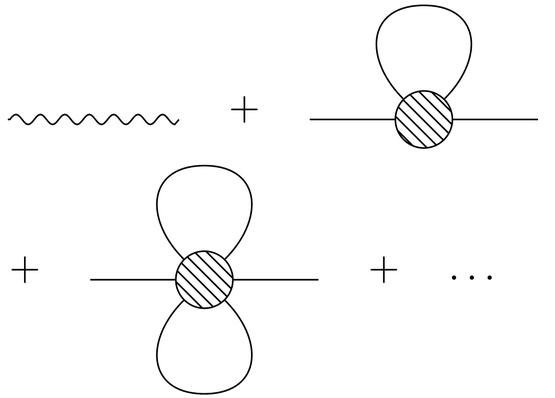
Decompose the linear power spectrum into a smooth (non-wiggly) and an oscillatory part

$$P_L(k) = P_L^{\rm nw}(k) + P_L^{\rm w}(k)$$

IR enhanced diagrams can be separated thanks to a power counting in the soft limit.

$$\mathcal{D}_{\mathbf{q}} P_L^{\rm w}(\mathbf{k}) = \frac{(\mathbf{q}\cdot\mathbf{k})}{q^2}\left[\exp\left(\mathbf{q}\cdot\nabla_{\mathbf{k}'}\right) - 1\right] P_L^{\rm w}(k')\Big|_{\mathbf{k}'=\mathbf{k}}$$

$P_{\delta\delta}^{\rm w,\,LO}(k)$ =



IR maximally enhanced diagrams:
daisy diagrams

...and can be consistently resummed to give LO and NLO power spectra

$$P_{\delta\delta}^{\rm w,LO}(\eta,k) = g^2(\eta)\, e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)$$

$$P_{\delta\delta}^{\rm NLO}(\eta,k) = g^2(\eta)\left[P_L^{\rm nw}(k) + \left(1+g^2(\eta)\,\mathcal{S}\right) e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)\right] + g^4(\eta)\, P_{\delta\delta}^{1-\rm loop}\left[P_L^{\rm nw}(k) + e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)\right]$$

Methodology: TSPT

[Blas et al., JCAP **07** (2016) 052]
[Blas et al., JCAP **07** (2016) 028]
[Vasudevan et al., JCAP **09** (2019) 037]

Decompose the linear power spectrum into a smooth (non-wiggly) and an oscillatory part

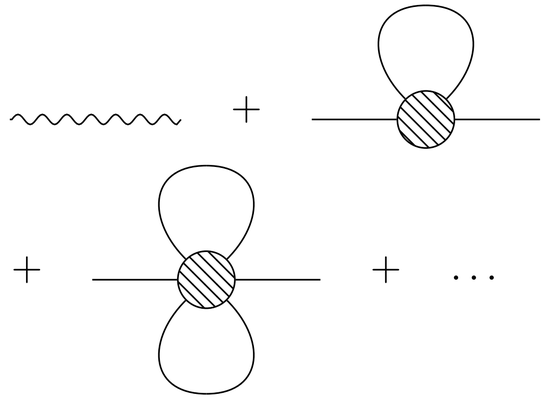
$$P_L(k) = P_L^{\rm nw}(k) + P_L^{\rm w}(k)$$

IR enhanced diagrams can be separated thanks to a power counting in the soft limit.

Translation operator

$$\mathcal{D}_{\mathbf{q}} P_L^{\rm w}(\mathbf{k}) = \frac{(\mathbf{q}\cdot\mathbf{k})}{q^2}\left[\exp\left(\mathbf{q}\cdot\nabla_{\mathbf{k}'}\right) - 1\right] P_L^{\rm w}(k')\Big|_{\mathbf{k}'=\mathbf{k}} \qquad \mathcal{T}_{\mathbf{q}}[\,\cdot\,] = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\mathbf{q}\cdot\nabla_{\mathbf{k}}\right)^n$$

$P_{\delta\delta}^{\rm w,\,LO}(k)$ =



IR maximally enhanced diagrams:
daisy diagrams

...and can be consistently resummed to give LO and NLO power spectra

$$P_{\delta\delta}^{\rm w,LO}(\eta,k) = g^2(\eta)\, e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)$$

$$P_{\delta\delta}^{\rm NLO}(\eta,k) = g^2(\eta)\left[P_L^{\rm nw}(k) + \left(1+g^2(\eta)\,\mathcal{S}\right)e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)\right] + g^4(\eta)\, P_{\delta\delta}^{1-\rm loop}\left[P_L^{\rm nw}(k) + e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)\right]$$

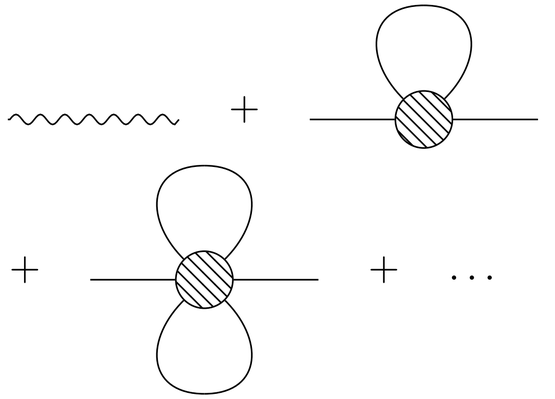
Methodology: TSPT

[Blas et al., JCAP **07** (2016) 052]
[Blas et al., JCAP **07** (2016) 028]
[Vasudevan et al., JCAP **09** (2019) 037]

Decompose the linear power spectrum into a smooth (non-wiggly) and an oscillatory part

$$P_L(k) = P_L^{\rm nw}(k) + P_L^{\rm w}(k)$$

$$P_{\delta\delta}^{\rm w,\,LO}(k) \quad = \quad + \quad + \quad + \quad \cdots$$



IR enhanced diagrams can be separated thanks to a power counting in the soft limit.

Translation operator

$$\mathcal{D}_{\mathbf{q}} P_L^{\rm w}(\mathbf{k}) = \frac{(\mathbf{q}\cdot\mathbf{k})}{q^2}\left[\exp\left(\mathbf{q}\cdot\nabla_{\mathbf{k}'}\right) - 1\right] P_L^{\rm w}(k')\Big|_{\mathbf{k}'=\mathbf{k}} \qquad \mathcal{T}_{\mathbf{q}}[\,\cdot\,] = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\mathbf{q}\cdot\nabla_{\mathbf{k}}\right)^n$$

IR maximally enhanced diagrams:
daisy diagrams

...and can be consistently resummed to give LO and NLO power spectra

How to compute them?

$$P_{\delta\delta}^{\rm w,LO}(\eta,k) = g^2(\eta)\, e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)$$

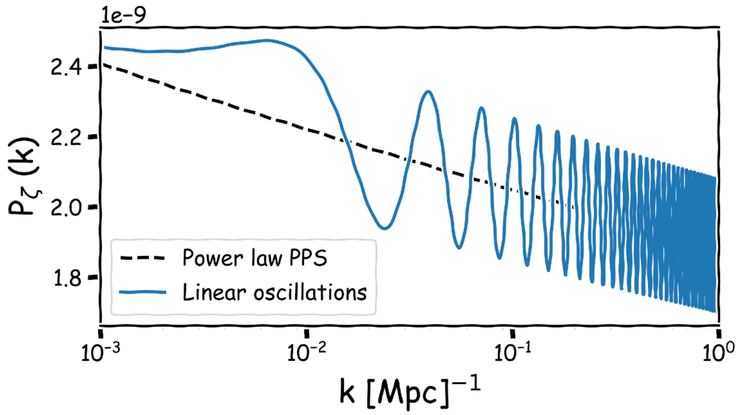
$$P_{\delta\delta}^{\rm NLO}(\eta,k) = g^2(\eta)\left[P_L^{\rm nw}(k) + \left(1+g^2(\eta)\mathcal{S}\right)e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)\right] + g^4(\eta)\, P_{\delta\delta}^{1-\rm loop}\left[P_L^{\rm nw}(k) + e^{-g^2(\eta)\mathcal{S}} P_L^{\rm w}(k)\right]$$

Linear features

1

Linear feature **template**:

$$\delta P_\zeta^{\rm lin}\left(k\right) = A_{\rm lin}\sin\left(\omega_{\rm lin}\frac{k}{k_*} + \phi_{\rm lin}\right)$$



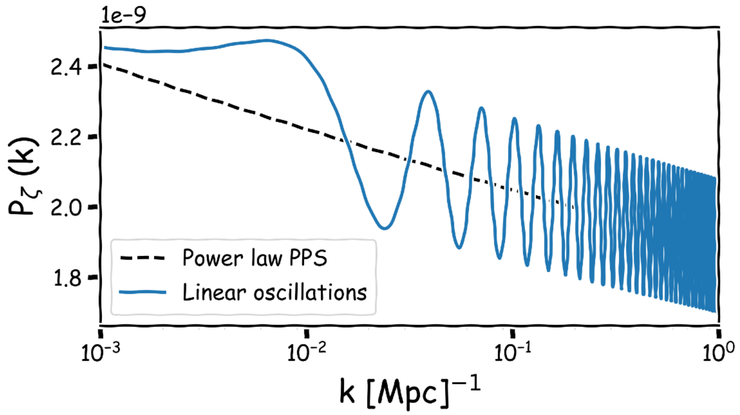
[Starobinsky, PRL **55** (1992) 489]
[Adams et al., PRD **64** (2001) 123514]
[Chen et al., JCAP **06** (2007) 023]
[Chen et al., JCAP **04** (2008) 010]

Linear features

1

Linear feature **template**:

$$\delta P_\zeta^{\rm lin}(k) = A_{\rm lin} \sin\left(\omega_{\rm lin}\frac{k}{k_*} + \phi_{\rm lin}\right)$$



[Starobinsky, PRL **55** (1992) 489]
[Adams et al., PRD **64** (2001) 123514]
[Chen et al., JCAP **06** (2007) 023]
[Chen et al., JCAP **04** (2008) 010]

Computation of **translation operator**

2

$$\mathbf{q}\cdot\nabla_{\mathbf{k}} e^{\pm i\omega_{\rm lin}k/k_*} = \pm i\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}e^{\pm i\omega_{\rm lin}k/k_*}$$

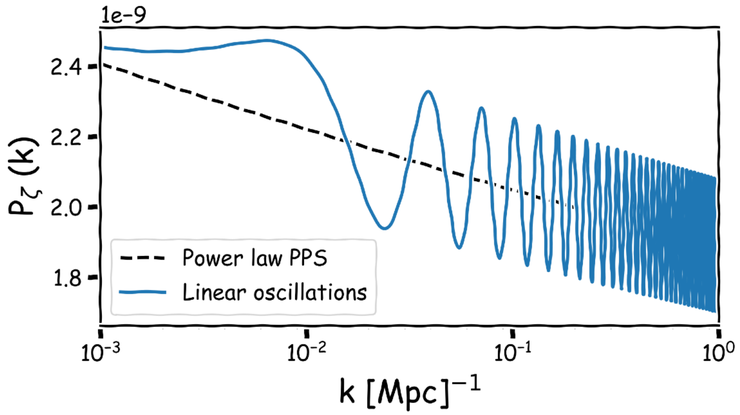
Linear features

[Starobinsky, PRL **55** (1992) 489]
[Adams et al., PRD **64** (2001) 123514]
[Chen et al., JCAP **06** (2007) 023]
[Chen et al., JCAP **04** (2008) 010]

1

Linear feature **template**:

$$\delta P_\zeta^{\rm lin}\left(k\right) = A_{\rm lin}\sin\left(\omega_{\rm lin}\frac{k}{k_*} + \phi_{\rm lin}\right)$$



Computation of **translation operator**

2

3

Computation of nonlinear damping

$$\mathbf{q}\cdot\nabla_{\mathbf{k}} e^{\pm i\omega_{\rm lin}k/k_*} = \pm i\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}e^{\pm i\omega_{\rm lin}k/k_*}$$

$$\mathcal{S}P_L^{\rm w}\left(k\right) = +\frac{1}{2}\int_{q\leq k_{\rm S}}\frac{{\rm d}^3q}{\left(2\pi\right)^3}\,P_L^{\rm nw}\left(q\right)\frac{\left(\mathbf{q}\cdot\mathbf{k}'\right)^2}{q^4}\left(\mathcal{T}_{\mathbf{q}}-1\right)\left(\mathcal{T}_{-\mathbf{q}}-1\right)P_L^{\rm w}\left(k'\right)\Big|_{\mathbf{k}'=\mathbf{k}}$$

$$= \int_{q\leq k_{\rm S}}\frac{{\rm d}^3q}{\left(2\pi\right)^3}\,P_L^{\rm nw}\left(q\right)\frac{\left(\mathbf{q}\cdot\mathbf{k}'\right)^2}{q^4}\left[1-\cosh\left(\mathbf{q}\cdot\nabla_{\mathbf{k}'}\right)\right]P_L^{\rm w}\left(k'\right)\Big|_{\mathbf{k}'=\mathbf{k}}$$

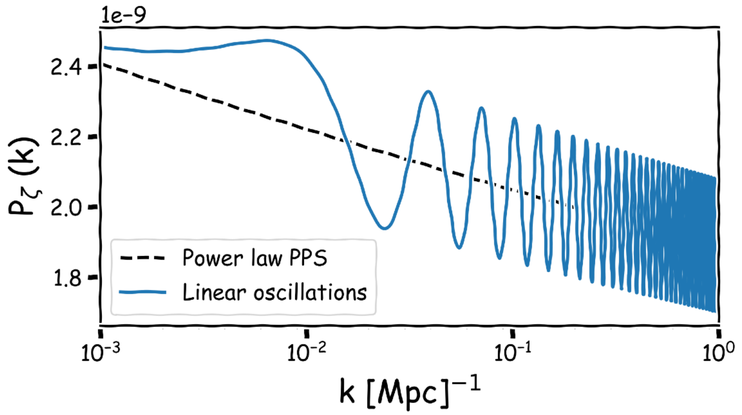
Linear features

[Starobinsky, PRL **55** (1992) 489]
[Adams et al., PRD **64** (2001) 123514]
[Chen et al., JCAP **06** (2007) 023]
[Chen et al., JCAP **04** (2008) 010]

1

Linear feature **template**:

$$\delta P_\zeta^{\rm lin}\left(k\right) = A_{\rm lin}\sin\left(\omega_{\rm lin}\frac{k}{k_*} + \phi_{\rm lin}\right)$$



2

Computation of **translation operator**

3

Computation of nonlinear damping

$$\mathbf{q}\cdot\nabla_{\mathbf{k}} e^{\pm i\omega_{\rm lin}k/k_*} = \pm i\frac{\omega_{\rm lin}}{k_*}\frac{\left(\mathbf{q}\cdot\mathbf{k}\right)}{k}e^{\pm i\omega_{\rm lin}k/k_*}$$

$$\begin{aligned}\mathcal{S}P_L^{\rm w}\left(k\right) &= +\frac{1}{2}\int_{q\leq k_{\rm S}}\frac{{\rm d}^3q}{\left(2\pi\right)^3}\,P_L^{\rm nw}\left(q\right)\frac{\left(\mathbf{q}\cdot\mathbf{k}'\right)^2}{q^4}\left(\mathcal{T}_{\mathbf{q}}-1\right)\left(\mathcal{T}_{-\mathbf{q}}-1\right)P_L^{\rm w}\left(k'\right)\Big|_{\mathbf{k}'=\mathbf{k}} \\ &= \int_{q\leq k_{\rm S}}\frac{{\rm d}^3q}{\left(2\pi\right)^3}\,P_L^{\rm nw}\left(q\right)\frac{\left(\mathbf{q}\cdot\mathbf{k}'\right)^2}{q^4}\left[1-\cosh\left(\mathbf{q}\cdot\nabla_{\mathbf{k}'}\right)\right]P_L^{\rm w}\left(k'\right)\Big|_{\mathbf{k}'=\mathbf{k}}\end{aligned}$$

...which finally gives:

$$\mathcal{S}P_L^{\rm w}\left(k\right) = P_L^{\rm w}\left(k\right)\frac{k^2}{6\pi^2}\int_0^{k_{\rm S}}{\rm d}q\,P_L^{\rm nw}\left(q\right)\left[1-j_0\left(\frac{\omega_{\rm lin}}{k_*}q\right)+2j_2\left(\frac{\omega_{\rm lin}}{k_*}q\right)\right] \equiv k^2\Sigma_{\rm lin}\left(k\right)P_L^{\rm w}\left(k\right)$$

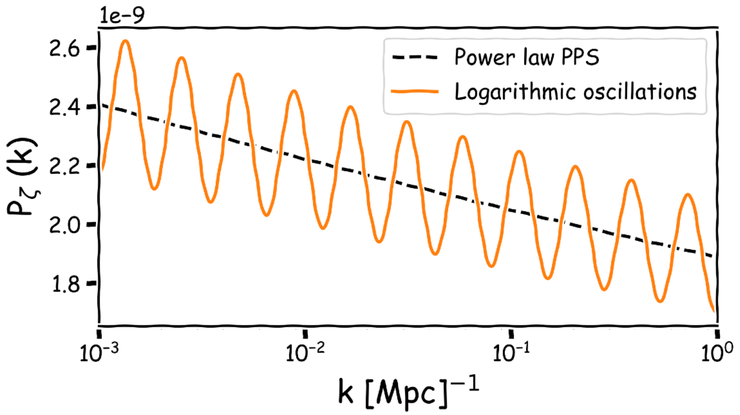
Logarithmic features

[Chen et al., JCAP **04** (2008) 010]
[Flauger et al., JCAP **06** (2010) 009]
[Flauger & Pajer, JCAP **01** (2011) 017]
[Chen, JCAP **12** (2010) 003]

1

Logarithmic feature **template**:

$$\delta P_\zeta^{\log}(k) = A_{\log} \sin\left(\omega_{\log} \ln \frac{k}{k_*} + \phi_{\log}\right)$$



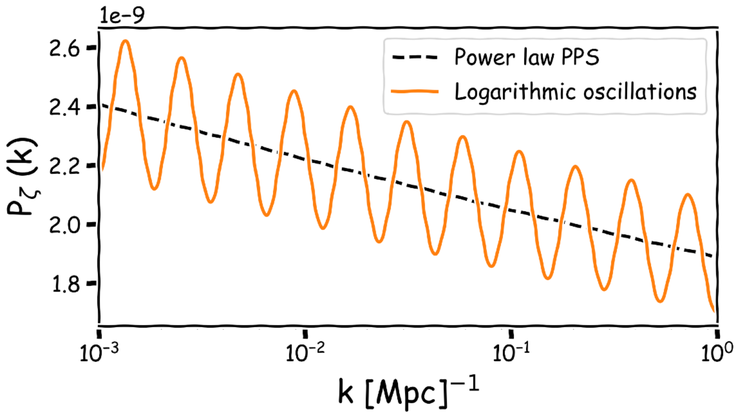
Logarithmic features

[Chen et al., JCAP **04** (2008) 010]
[Flauger et al., JCAP **06** (2010) 009]
[Flauger & Pajer, JCAP **01** (2011) 017]
[Chen, JCAP **12** (2010) 003]

1

Logarithmic feature **template**:

$$\delta P_\zeta^{\log}(k) = A_{\log} \sin\left(\omega_{\log} \ln \frac{k}{k_*} + \phi_{\log}\right)$$



Computation of **translation operator**

2

$$\mathbf{q} \cdot \nabla_{\mathbf{k}} e^{\pm i \omega_{\rm lin} k/k_*} = \pm i \frac{\omega_{\rm lin}}{k_*} \frac{(\mathbf{q} \cdot \mathbf{k})}{k} e^{\pm i \omega_{\rm lin} k/k_*}$$

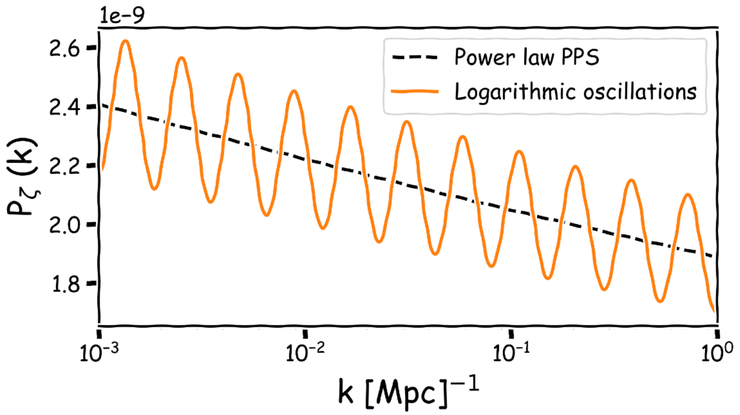
Logarithmic features

[Chen et al., JCAP **04** (2008) 010]
[Flauger et al., JCAP **06** (2010) 009]
[Flauger & Pajer, JCAP **01** (2011) 017]
[Chen, JCAP **12** (2010) 003]

1

Logarithmic feature **template**:

$$\delta P_\zeta^{\log}(k) = A_{\log} \sin\left(\omega_{\log} \ln \frac{k}{k_*} + \phi_{\log}\right)$$



2

Computation of **translation operator**

$$\mathbf{q}\cdot\nabla_{\mathbf{k}} e^{\pm i\omega_{\rm lin}k/k_*} = \pm i\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}e^{\pm i\omega_{\rm lin}k/k_*}$$

What happens adding more steps?

$$\mathcal{T}_{\mathbf{q}}\left[e^{\pm i\omega_{\log}\ln(k/k_*)}\right] = \sum_{n=0}^{\infty}(\mathbf{q}\cdot\nabla_{\mathbf{k}})^{(n-1)}\left[\pm i\omega_{\log}\frac{(\mathbf{q}\cdot\mathbf{k})}{k^2}e^{\pm i\omega_{\log}\ln(k/k_*)}\right]$$

$$= \sum_{n=0}^{\infty}(\mathbf{q}\cdot\nabla_{\mathbf{k}})^{(n-2)}\left\{\pm i\omega_{\log}\left[\pm i\omega_{\log}\frac{(\mathbf{q}\cdot\mathbf{k})^2}{k^4} + \frac{q^2}{k^2} - 2\frac{(\mathbf{q}\cdot\mathbf{k})^2}{k^4}\right]e^{\pm i\omega_{\log}\ln(k/k_*)}\right\}$$

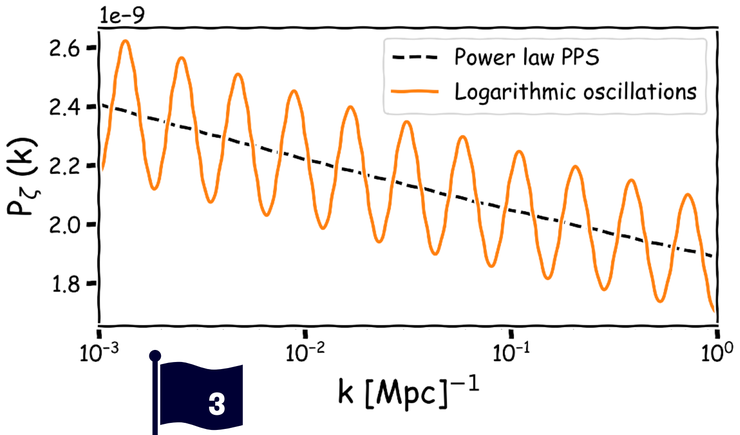
Logarithmic features

[Chen et al., JCAP **04** (2008) 010]
[Flauger et al., JCAP **06** (2010) 009]
[Flauger & Pajer, JCAP **01** (2011) 017]
[Chen, JCAP **12** (2010) 003]

1

Logarithmic feature **template**:

$$\delta P_\zeta^{\log}(k) = A_{\log} \sin\left(\omega_{\log} \ln \frac{k}{k_*} + \phi_{\log}\right)$$



3

Computation of nonlinear damping (strong approximation)

$$\mathcal{S}P_L^{\mathrm{w}}(k) = P_L^{\mathrm{w}}(k) \frac{k^2}{6\pi^2} \int_0^{k_{\mathrm{S}}} \mathrm{d}q\, P_L^{\mathrm{nw}}(q) \left[1 - j_0\left(\frac{\omega_{\log}}{k} q\right) + 2 j_2\left(\frac{\omega_{\log}}{k} q\right)\right] \equiv k^2 \Sigma_{\log}(k)\, P_L^{\mathrm{w}}(k)$$

2

Computation of **translation operator**

$$\mathbf{q} \cdot \nabla_{\mathbf{k}} e^{\pm i \omega_{\mathrm{lin}} k/k_*} = \pm i \frac{\omega_{\mathrm{lin}}}{k_*} \frac{(\mathbf{q}\cdot\mathbf{k})}{k} e^{\pm i \omega_{\mathrm{lin}} k/k_*}$$

What happens adding more steps?

$$\mathcal{T}_{\mathbf{q}}\left[e^{\pm i\omega_{\log}\ln(k/k_*)}\right] = \sum_{n=0}^{\infty} (\mathbf{q}\cdot\nabla_{\mathbf{k}})^{(n-1)} \left[\pm i\omega_{\log} \frac{(\mathbf{q}\cdot\mathbf{k})}{k^2} e^{\pm i\omega_{\log}\ln(k/k_*)}\right]$$

$$= \sum_{n=0}^{\infty} (\mathbf{q}\cdot\nabla_{\mathbf{k}})^{(n-2)} \left\{\pm i\omega_{\log}\left[\pm i\omega_{\log}\frac{(\mathbf{q}\cdot\mathbf{k})^2}{k^4} + \frac{q^2}{k^2} - 2\frac{(\mathbf{q}\cdot\mathbf{k})^2}{k^4}\right] e^{\pm i\omega_{\log}\ln(k/k_*)}\right\}$$

Mixed oscillatory terms

Addition of primordial oscillatory features generates interference with BAO

$$P^{\rm w}\left(k\right) \equiv P^{\rm w}_{\rm BAO}\left(k\right) + P^{\rm w}_{X}\left(k\right) + P^{\rm w}_{\rm BAO}\left(k\right)\delta P^{X}_{\zeta}\left(k\right)$$

Thanks to trigonometric relations we can describe it as a superposition of

$$P^{\rm w}_{\rm BAO}\left(k\right)\delta P^{\rm lin}_{\zeta}\left(k\right) = \frac{1}{2}A_{\rm BAO}\left(k\right)\left[P^{\rm w}_{\rm lin}\left(k\right)\big|^{\varphi_{\rm lin}+\pi/2}_{\omega_{\rm lin}-k_{*}\omega_{\rm BAO}} - P^{\rm w}_{\rm lin}\left(k\right)\big|^{\varphi_{\rm lin}+\pi/2}_{\omega_{\rm lin}+k_{*}\omega_{\rm BAO}}\right]$$

Computation of dampings mimics what happens for single oscillatory terms. Subtleties arise for the logarithmic case, but still allow for a consistent description.

$$\begin{aligned}\mathcal{S}\left[P^{\rm w}_{\rm BAO}\left(k\right)\delta P^{\rm lin}_{\zeta}\left(k\right)\right] &= \\ &= \frac{k^2}{2}A_{\rm BAO}\left(k\right)\left[\Sigma_{\rm lin}\left(k\right)P^{\rm w}_{\rm lin}\left(k\right)\big|^{\varphi_{\rm lin}+\pi/2}_{\omega_{\rm lin}-k_{*}\omega_{\rm BAO}} - \Sigma_{\rm lin}\left(k\right)P^{\rm w}_{\rm lin}\left(k\right)\big|^{\varphi_{\rm lin}+\pi/2}_{\omega_{\rm lin}+k_{*}\omega_{\rm BAO}}\right]\end{aligned}$$

Mixed oscillatory terms

Addition of primordial oscillatory features generates interference with BAO

$$P^{\rm w}(k) \equiv P^{\rm w}_{\rm BAO}(k) + P^{\rm w}_{X}(k) + P^{\rm w}_{\rm BAO}(k)\,\delta P^{X}_{\zeta}(k)$$

Usually **neglected** because subdominant. It misses a **specific characterisation** of its impact

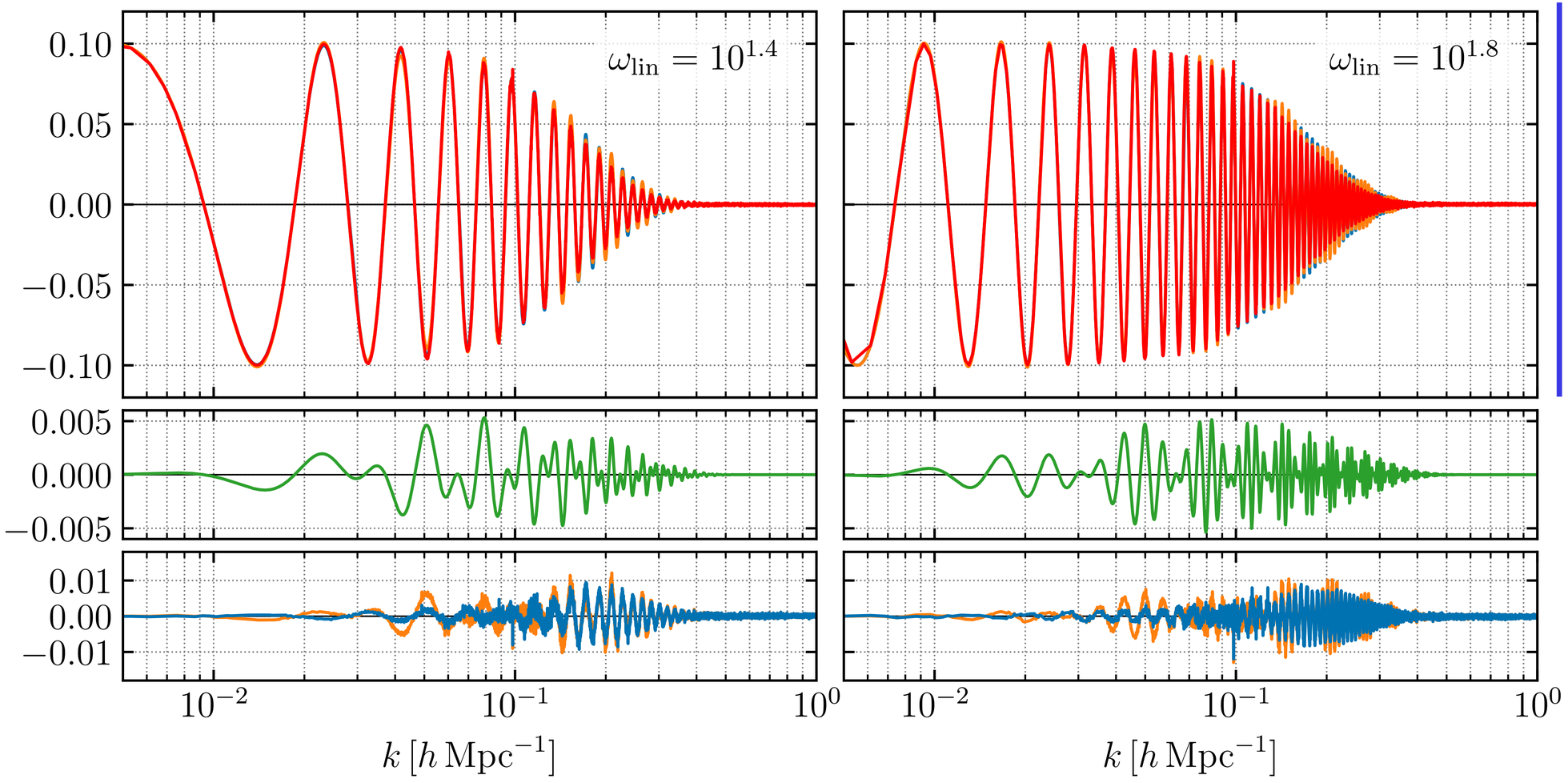
Thanks to trigonometric relations we can describe it as a superposition of

$$P^{\rm w}_{\rm BAO}(k)\,\delta P^{\rm lin}_{\zeta}(k) = \frac{1}{2} A_{\rm BAO}(k) \left[P^{\rm w}_{\rm lin}(k)\big|^{\varphi_{\rm lin}+\pi/2}_{\omega_{\rm lin}-k_*\omega_{\rm BAO}} - P^{\rm w}_{\rm lin}(k)\big|^{\varphi_{\rm lin}+\pi/2}_{\omega_{\rm lin}+k_*\omega_{\rm BAO}} \right]$$

Computation of dampings mimics what happens for single oscillatory terms. Subtleties arise for the logarithmic case, but still allow for a consistent description.

$$\begin{aligned}\mathcal{S}\left[P^{\rm w}_{\rm BAO}(k)\,\delta P^{\rm lin}_{\zeta}(k)\right] &= \\ &= \frac{k^2}{2} A_{\rm BAO}(k) \left[\Sigma_{\rm lin}(k)\, P^{\rm w}_{\rm lin}(k)\big|^{\varphi_{\rm lin}+\pi/2}_{\omega_{\rm lin}-k_*\omega_{\rm BAO}} - \Sigma_{\rm lin}(k)\, P^{\rm w}_{\rm lin}(k)\big|^{\varphi_{\rm lin}+\pi/2}_{\omega_{\rm lin}+k_*\omega_{\rm BAO}} \right]\end{aligned}$$

Mixed oscillatory terms



Difference between matter power spectrum (with primordial linear oscillations) and power-law PPS divided by the non-wiggly power spectrum (upper panels)

ratio of the NLO matter power spectrum with mixed term to that without (middle panels)

Differences between the theoretical spectra at NLO, and the simulations (lower panel).

Scale dependent amplitudes

Oscillatory templates are enriched with scale dependent amplitudes…

$$A(k) = \begin{cases} A_{\rm X}^{\rm WP} \exp\left[-(k-\mu)^2/2\Delta^2\right] \\ A_{\rm X}^{\rm PL}\,(k/k_\star)^n \end{cases}$$

A lot of models (the same BAO) present this feature, Usually neglected because suppressed in the power counting

A **new damping factor** appears and we need new formulas to take it into account properly

…new features appear in the computation of the spectra.

$$\begin{aligned}
&\left[1-\cosh\left(\mathbf{q}\cdot\nabla_{\mathbf{k}}\right)\right] P^{\rm w}_{\rm WP,lin}(k) = \\
&\quad = \left\{1-\cos\left[\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\cosh\left[\frac{(k-\mu)}{\Delta^2}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\right\} P^{\rm w}_{\rm WP,lin}(k) \\
&\qquad + \sin\left[\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\sinh\left[\frac{(k-\mu)}{\Delta^2}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right] P^{\rm w}_{\rm WP,lin}(k)\big|_{\phi+\pi/2}
\end{aligned}$$

$$\begin{aligned}
P^{\rm w,LO}_{\delta\delta}(\eta,k) = g^2(\eta)\Big\{ & e^{-g^2(\eta)k^2\Sigma^{\rm WP}_{\rm lin}(k)}\cos\left[g^2(\eta)\,k^2\hat{\Sigma}^{\rm WP}_{\rm lin}(k)\right] P^{\rm w}_{\rm WP,lin}(k) \\
& - e^{-g^2(\eta)k^2\Sigma^{\rm WP}_{\rm lin}(k)}\sin\left[g^2(\eta)\,k^2\hat{\Sigma}^{\rm WP}_{\rm lin}(k)\right] P^{\rm w}_{\rm WP,lin}(k)\big|_{\phi+\pi/2}\Big\}
\end{aligned}$$

Scale dependent amplitudes

Oscillatory templates are enriched with scale dependent amplitudes…

$$A(k) = \begin{cases} A_{\rm X}^{\rm WP} \exp\left[-(k-\mu)^2/2\Delta^2\right] \\ A_{\rm X}^{\rm PL}\,(k/k_\star)^n \end{cases}$$

A lot of models (the same BAO) present this feature, Usually neglected because suppressed in the power counting

A **new damping factor** appears and we need new formulas to take it into account properly

…new features appear in the computation of the spectra.

$$\begin{aligned}
&\left[1-\cosh\left(\mathbf{q}\cdot\nabla_{\mathbf{k}}\right)\right] P^{\rm w}_{\rm WP,lin}(k) = \\
&\quad = \left\{1-\cos\left[\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\cosh\left[\frac{(k-\mu)}{\Delta^2}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\right\} P^{\rm w}_{\rm WP,lin}(k) \\
&\qquad + \sin\left[\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\sinh\left[\frac{(k-\mu)}{\Delta^2}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right] P^{\rm w}_{\rm WP,lin}(k)\Big|_{\phi+\pi/2}
\end{aligned}$$

Contribution from an **out-of-phase** power spectrum

$$\begin{aligned}
P^{\rm w,LO}_{\delta\delta}(\eta,k) = g^2(\eta)\Big\{ & e^{-g^2(\eta)k^2\Sigma^{\rm WP}_{\rm lin}(k)}\cos\left[g^2(\eta)\,k^2\hat{\Sigma}^{\rm WP}_{\rm lin}(k)\right] P^{\rm w}_{\rm WP,lin}(k) \\
& - e^{-g^2(\eta)k^2\Sigma^{\rm WP}_{\rm lin}(k)}\sin\left[g^2(\eta)\,k^2\hat{\Sigma}^{\rm WP}_{\rm lin}(k)\right] P^{\rm w}_{\rm WP,lin}(k)\Big|_{\phi+\pi/2}\Big\}
\end{aligned}$$

Scale dependent amplitudes

Oscillatory templates are enriched with scale dependent amplitudes…

$$A(k) = \begin{cases} A_{\rm X}^{\rm WP} \exp\left[-(k-\mu)^2/2\Delta^2\right] \\ A_{\rm X}^{\rm PL}\,(k/k_\star)^n \end{cases}$$

A lot of models (the same BAO) present this feature, Usually neglected because suppressed in the power counting

A **new damping factor** appears and we need new formulas to take it into account properly

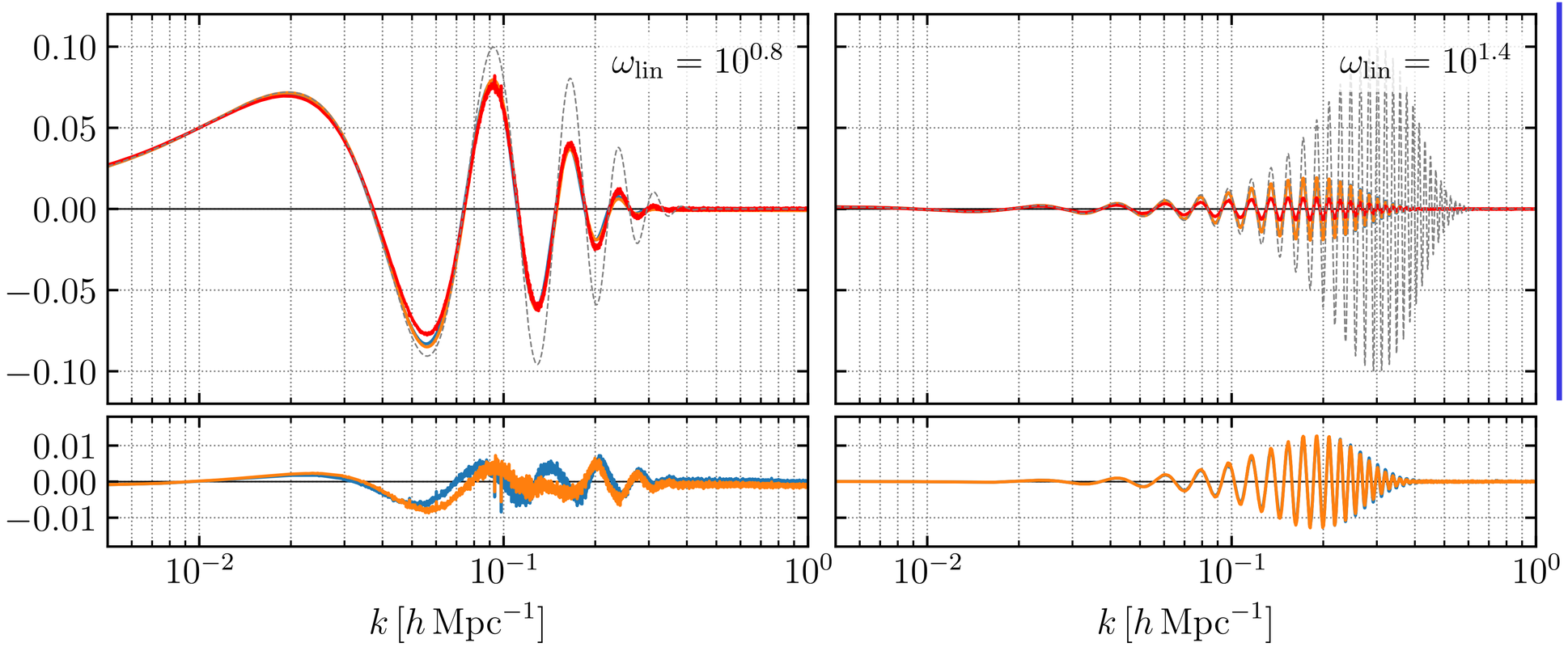
…new features appear in the computation of the spectra.

$$\begin{aligned}
&\left[1-\cosh\left(\mathbf{q}\cdot\nabla_{\mathbf{k}}\right)\right] P^{\rm w}_{\rm WP,lin}(k) = \\
&\quad = \left\{1-\cos\left[\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\cosh\left[\frac{(k-\mu)}{\Delta^2}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\right\} P^{\rm w}_{\rm WP,lin}(k) \\
&\qquad + \sin\left[\frac{\omega_{\rm lin}}{k_*}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right]\sinh\left[\frac{(k-\mu)}{\Delta^2}\frac{(\mathbf{q}\cdot\mathbf{k})}{k}\right] P^{\rm w}_{\rm WP,lin}(k)\big|_{\phi+\pi/2}
\end{aligned}$$

Contribution from an **out-of-phase** power spectrum

$$\begin{aligned}
P^{\rm w,LO}_{\delta\delta}(\eta,k) = g^2(\eta)\Big\{ & e^{-g^2(\eta)k^2\Sigma^{\rm WP}_{\rm lin}(k)} \cos\left[g^2(\eta)\,k^2\hat{\Sigma}^{\rm WP}_{\rm lin}(k)\right] P^{\rm w}_{\rm WP,lin}(k) \\
& - e^{-g^2(\eta)k^2\Sigma^{\rm WP}_{\rm lin}(k)} \sin\left[g^2(\eta)\,k^2\hat{\Sigma}^{\rm WP}_{\rm lin}(k)\right] P^{\rm w}_{\rm WP,lin}(k)\big|_{\phi+\pi/2}\Big\}
\end{aligned}$$

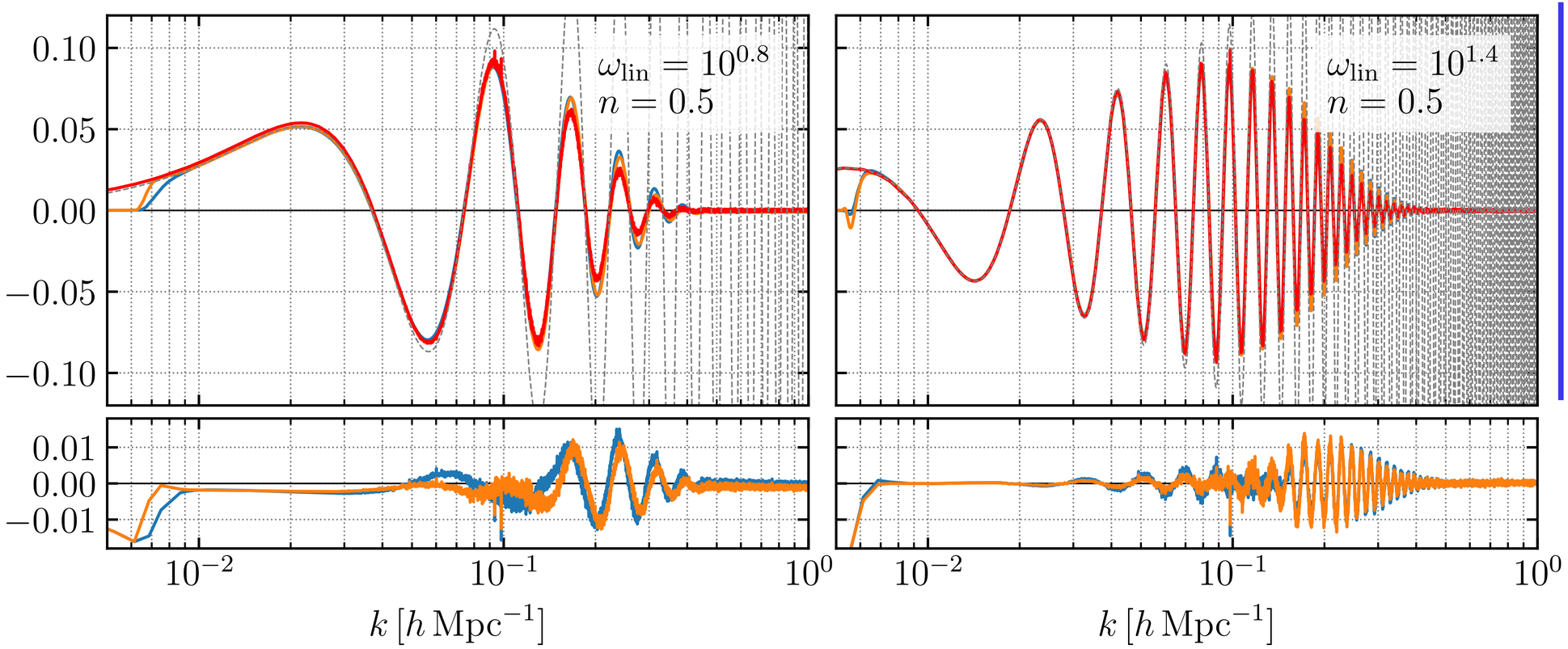
Scale dependent amplitudes



Difference between matter power spectrum (with primordial linear oscillations) and power-law PPS divided by the non-wiggly power spectrum (upper panels)

Differences between the theoretical spectra at LO and NLO, and the simulations (lower panel).

Scale dependent amplitudes



Difference between matter power spectrum (with primordial linear oscillations) and power-law PPS divided by the non-wiggly power spectrum (upper panels)

Differences between the theoretical spectra at LO and NLO, and the simulations (lower panel).

Scale dependent amplitudes

As scale dependence increases, the amplitude corrections grow larger.

For extreme values of the envelope parameters this can spoil the pour counting defined in our methodology, making the whole computation unreliable.

$$\Sigma_{\rm lin}^{\rm WP}\left(k\right)=\Sigma_{\rm lin}\left(k\right)+\frac{1}{\Delta^4}\delta\Sigma_{\rm lin}^{(4)}\left(k\right)+\mathcal{O}\left(\frac{1}{\Delta^8}\right),$$

$$\hat{\Sigma}_{\rm lin}^{\rm WP}\left(k\right)=\frac{1}{\Delta^2}\delta\hat{\Sigma}_{\rm lin}^{(2)}\left(k\right)+\mathcal{O}\left(\frac{1}{\Delta^6}\right)$$

Check that **relative corrections** to the power spectrum does not exeed a small percentage of it

$$\frac{P_{\delta\delta}^{\rm w,LO}\left(k\right)-P_{\delta\delta}^{\rm w,LO}\left(k\right)\Big|_{\Delta\to\infty}}{P_{\delta\delta}^{\rm w,LO}\left(k\right)\Big|_{\Delta\to\infty}}=\frac{k^2}{\Delta^2}\delta\hat{\Sigma}_{\rm lin}^{(2)}\left(k\right)\frac{P_L^{\rm w}\left(k\right)|_{\phi+\pi/2}}{P_L^{\rm w}\left(k\right)}$$

Leading corrections to the damping factors

$$\delta\hat{\Sigma}_{\rm lin}^{(2)}\left(k\right)\equiv\frac{1}{6\pi^2}\int_0^{k_{\rm S}}{\rm d}q\;P_{\rm nw}\left(q\right)\frac{3\left(k-\mu\right)}{q^3\tilde{\omega}_{\rm lin}^4}$$
$$\times\left[q\tilde{\omega}_{\rm lin}\left(-6+q^2\tilde{\omega}_{\rm lin}^2\right)\cos\left(q\tilde{\omega}_{\rm lin}\right)-3\left(-2+q^2\tilde{\omega}_{\rm lin}^2\right)\sin\left(q\tilde{\omega}_{\rm lin}\right)\right]$$

...where $\tilde{\omega}_{\rm lin}\equiv\omega_{\rm lin}/k_*$ for simplicity.

Conclusions

- We have applied TSPT formalism to compute LO and NLO matter power spectrum for non-standard effects as features in the PPS of scalar perturbations;
- We include to the standard description the mixed term between features and BAO, and the contribution of scale dependent amplitudes refining the modelling of nonlinear damping;
- All the results have been tested against dark matter simulations. We found them in agreement for all those scales relevant for galaxy clustering analysis $k < 0.3\ h\,\mathrm{Mpc}^{-1}$;
- We aim to perform a precise validation of our results with realistic simulations for upcoming LSS surveys, such as DESI and Euclid.

New Physics from Galaxy Clustering III

Thank you!

Refining the nonlinear modelling of primordial oscillatory features

NICOLA BARBIERI (UNIFE) – 04/11/2024